ERRATUM

Graphs for which the least eigenvalue is minimal, II

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Volume 429, pages 2168–2179

The authors are indebted to Miroslav Petrović for pointing out an error in the proof of Proposition 4.2. Consider the graphs H_m as m increases from $t(n-t)+1$ to $(t+1)(n-t-1)$. It is necessary to admit the possibility that all H_m are bipartite. In the contrary situation, it remains the case that H_m is first non-bipartite and then bipartite, with either possibility admitted at the point of transition. However, the point of transition is not necessarily at $t(n-t)+1$. Accordingly, Theorem 4.1 should be reformulated as follows:

Theorem 4.1. *Let G be a graph whose least eigenvalue is minimal among the connected graphs of order n and size m. Then*

(i) if $m = t(n-t)$ for $t \in \{1, 2, \ldots, \lfloor \frac{n}{2} \rfloor\}$, then $G = K_{t,n-t}$;

(ii) if $t(n-t) < m < (t+1)(n-t-1)$ for some $t \in \{1, 2, \ldots, \lfloor \frac{n}{2} \rfloor - 1\}$, then there exists an integer s such that $t(n-t) < s < (t+1)(n-t-1)$, G is non-bipartite whenever $t(n-t) < m < s$, and G is bipartite whenever $s < m < (t+1)(n-t-1)$;

(iii) if $\lfloor \frac{n}{2} \rfloor \lceil \frac{n}{2} \rceil < m < \binom{m}{2}$ then G is non-bipartite and hence the join of two nested split graphs.

The following accounts for the phenomenon detailed in Theorem 4.1(ii).

Proposition 4.2. *Suppose that $t(n-t) < m < (t+1)(n-t-1)$ for some $t \in \{1, 2, \ldots, \lfloor \frac{n}{2} \rfloor - 1\}$. If some graph H_m is bipartite then every graph H_{m+1} is bipartite.*

Proof. Suppose by way of contradiction that H_m is bipartite and H_{m+1} is non-bipartite. Let $\mathbf{x} = (x_1, x_2, \ldots, x_n)^T$ be a unit eigenvector of $H = H_{m+1}$ corresponding to $\lambda(H)$. From Proposition 1.2, we know that H contains an edge $e = vw$ such that $x_v x_w \geq 0$ and $H - e$ is connected. Writing $H^* = H - e$, we have

$$\lambda(H^*) \leq \mathbf{x}^T A_{H^*} \mathbf{x} = \mathbf{x}^T A_H \mathbf{x} - 2x_v x_w \leq \mathbf{x}^T A_H \mathbf{x} = \lambda(H).$$

Since H_m is bipartite we have

$$\lambda(G_m) = \lambda(H_m) \leq \lambda(H^*) \leq \lambda(H_{m+1}) \leq \lambda(G_{m+1}).$$

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On the other hand we have $\lambda(G_{m+1}) < \lambda(G_m)$ by Lemma 3.2. This contradiction completes the proof. $\square$

Fig. 2 shows the behaviour of $\lambda(H_m)$ when $n = 9$. Finally, Proposition 4.4 should be recast as follows, with essentially the same proof.

Proposition 4.4. *If H_m is non-bipartite and $m = t(n-t)+1$ where $t \in \{1, 2, \ldots, \lfloor \frac{n}{2} \rfloor - 1\}$ then $H_m = K_{t,n-t} + e$, where e is an edge joining two vertices of degree t in $K_{t,n-t}$.*